

Collaborative Learning in Undergraduate Mathematics

Practical Issues for Implementation, and Strategies that work

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What this talk will not be about!

- I will not be showing any studies or experiments that gives evidence that Collaborative Learning works.
 - Suffices to say that Collaborative learning as a philosophy and technique of interaction is rooted in constructivist and social learning theories as well as the pedagogy of social justice
- I will not be justifying if/why this is a better form of learning than traditional form of lecture or other forms of Group Work.
 - I will not be distinguishing between cooperative learning and collaborative learning.
 - But I will be highlighting some key features that distinguishes CL from “traditional” group work.

Collaborative/Cooperative Learning

- Both GW and CL incorporates different perspectives, experiences, knowledge, and skill sets
- Working together to complete an assigned task is not necessarily the same as collaborating to learn!

“Cooperative learning is a highly structured form of group work that focuses on the problem solving that - when directed by an effective teacher - can lead to deep learning, critical thinking, and genuine paradigm shifts in students’ thinking.” (Millis, 2010)

Five Vital Elements for Productive Cooperation (Johnson & Johnson, 1987; Johnson et al., 1998)

1. Positive Interdependence

- Each student should **perceive** that they are linked with others and cannot succeed unless others do.
- **Possible mechanisms** - joint rewards (for all group members), shared resources (e.g. only one worksheet for the group), and roles (e.g. reporter, elaborator, skeptic, etc.)
 - CLIC/CLOC has Manager + Spokesperson + Strategist

2. Individual Accountability

- Learn together with the goal of subsequently performing individually.
- No “Social Loafing”; each student must **earn** their grade and be individually measurable.
- **Possible mechanisms** - Ask for an explanation or presentation of work; individual assessments e.g. quizzes.



Five Vital Elements for Productive Cooperation

3. Positive/Promotive Interactions

- Forge meaningful, face-to-face interactions (two to four members), students promote each other's success through helping, supporting, encouraging, and praising one another's efforts.
- **Possible mechanisms** -
 - Monitor interactions and have pre-prepared "Scripts" to use when you encounter incivility in the classroom.
 - Provide examples of proper language for Academic discourse. [Kate Kinsella, 2017, [PDF](#)]

4. Social skills

- interpersonal skills needed to work effectively with others (leadership, trust-building, conflict management, etc.) are learned behaviors – need to be taught alongside academic skills.

5. Group processing

- Metacognition exercises - Take time to analyze the group's processes and identify possible improvements to become more efficient, more effective at working together.
- **Possible mechanisms** - Dedicated time for presentations, Minute papers at the end of session, using individual role sheets

Comparison of collaborative learning vs. group work

All collaborative learning is done in a group, but not all group work is inherently collaborative!
([Scheuermann](#), 2018)

COLLABORATIVE LEARNING

- Group effort required
- Learners accountable to each other
- Social skills are improved
- Helping and sharing is expected
- Emphasis on process and product

“TRADITIONAL” GROUP WORK

- “Divide and Conquer” mentality
- More free-riders
- Minimal interaction required
- Helping and sharing is minimal
- Emphasis on product only

Challenges

Challenge #1 [Issues related to Interdependence]

- **Lack of Experience**
- **Lack of Information/Guidance**
- **Lack of role management** → Leaders and Slackers

Consequence:

- (some) Students believe that faculty lack effective communication skills, doubt that faculty could or should teach students how to work in groups.
- Often frustrated as they perceive instructors are shirking their responsibilities

Challenge #2 [Issues related to Group and Individual Accountability]

- **Lack of Emphasis**
- **Lack of Assessment of Communication skills**

Discussion Question:

- How do we (and should we) assess the development of group communication skills? More on this at the end.

Challenge #3 [Increased class preparation time]

- Needs extra time to construct appropriate questions.

Counterpoint:

However, just like lecture material, these activities can be used year upon year with little extra effort. The main hurdle for instructors new to the idea is the development of teaching habits.

Challenge #4 [Issue Related to Positive interaction and Maintaining a constructive environment]

- **Set the expectations.**
- **Sell it!** Mention some research to back up your process
- **Implement only after careful consideration!** Try one or two learning targets first → TAPPS
- **Prepare scripts ahead of time.** Will you condemn them on the spot or just pull them to the side?
 - What will you say to a student who consistently shows up late to group meetings?
 - What will you say to a student who is adamant about not participating - arguing that they learn better by working alone?
 - What will you say to a student who is dismissive of what their peers contribute to the group or ostracize a group member?
 - What will you say to a student who is bullying/harassing or is just racist to a peer?

Challenges and Resistance specifically in Math (#5)

- Math is Objective
 - leaves little room for debate
 - This rigour potentially hinders the possibility for open discussion and certainly opinionated debate. When one side is objectively wrong, it takes away the fun.

Possible Fix:

- Construct mathematical problems that, for example, admit multiple approaches to the correct solution, or can be viewed from different perspectives.
- A lot on this later.

Challenges and Resistance specifically in Math (#6)

- Math is Abstract
 - Sometimes, it just takes time and deep, undistracted thought.
 - Jumping straight into CL activities upon first sight of new material can be overwhelming for students that are used to learning the material carefully in their own time.
 - Some group activities require the ability to discuss material in abstract terms, something potentially off-putting for inexperienced students.

Counterpoint:

- This may dissuade students from discussing Math verbally, however given the importance of this skill in research and industry, what better way to practise this skill than through cooperative learning exercises?

Challenges and Resistance specifically in Math (#7)

- Math is Sequential
 - Math syllabi are organised so that material does not appear before the necessary prerequisites
 - Stresses the lecturers to cover all the material in the syllabus, exerting a time pressure. No luxury of cutting syllabus material to provide extra time for cooperative learning activities.
 - Some faculty may be concerned that instruction in group skills will take precious time away from teaching discipline-specific content and some are unsure of their own abilities to teach students how to collaborate effectively.

Contd.

Possible Fix:

- construct group exercises that cover material NOT covered in lecture time
 - for example use a group exercise to further provide examples of review of a topic that will be used extensively in later lectures.
 - Alternatively, students can be held responsible for covering some of the material in their own time (e.g. flipped classroom). This material can then be used in the cooperative learning exercises, to ensure it has been understood.
- Integrate instruction about group dynamics along with instruction about content knowledge in introductory courses, students are likely to carry the lessons learned into subsequent courses.

Practical Implementation Considerations

Some Practical Implementation Considerations

- Physical layout of the Classroom
- Student Orientation to CL
 - Create community-building group activities that are Mathematical in nature.
 - E.g. Ask them to guesstimate a real-life quantity, let students introduce each others' favorite mathematical fact or formula, give students questions ahead of time so that they can “raise their hands”.
 - A new experience for some - expect both active and passive resistance if not prepared. Ensure that they understand it is “mandatory” and integral part of course - most student respond to clear statements of how it affects their grade.
 - Ask students to create group behavior norms themselves. → Metacognition

Some Practical Implementation Considerations contd.

- Duration of Groups
 - Permanent groups are suggested - except perhaps the first one or two weeks.
 - Develops camaraderie, reduces scheduling issues, tends to improve attendance.
- Formation Techniques vary
 - Random/Pseudo-random - good for first weeks to produce heterogeneity, but possibly skewed ability, or personality conflict, incompatible schedule
 - Instructor selected - needs to consider several factors, not just their grades
 - DEI considerations, schedule, commute
 - Student selected - set parameters
 - E.g. do not allow all students from the same foreign country to congregate, make students do it in class where they write down their own strengths and weaknesses, check with the shy/passive students personally
- Monitoring groups
 - Reinforce desired behaviors by circulating through the room and privately/publicly praising group members, solicit journal entries e.g. attendance record, percentage of time a student talked during the meeting (role-sheets)

Cooperative Strategies for Day-to-day Usage

Cooperative Strategies for Day-to-day Usage

1. Think-Pair-Share or Think-Aloud Pair Problem Solving (Brent and Felder, 2016)

- Involves students working together in pairs (or small groups) with a handout that contains both course material and problem solving activities.
- Cover course content AND implement active learning, → appealing for syllabus-heavy courses.

Usage

- Brainstorm, discuss concepts, provide foundation for later class.

Examples

- If $f'(c)=0$, what can you say about the function at $(c,f(c))$?
- Express a model as a ODE.
- If a and b are elements in group that have order 2 and 3, is there necessarily an element of order 6?

Cooperative Strategies for Day-to-day Usage contd.

2. Think-Share-Write

- No/minimal writing during individual thinking time
- Emphasis on individual assessment

Usage

- At beginning check on homework, at end check for understanding of the day, organize thoughts and improve math notation usage

Examples:

- When constructing an induction proof, it is necessary to find a base case. Explain what it means and why it's necessary.
- Every subspace of \mathbb{R}^3 will contain the origin. Write a proof.
- State two definition of compactness and prove they are equivalent.

Cooperative Strategies for Day-to-day Usage contd.

3. Data Sharing

- Applicable for Math Modeling or for generating a large amount of data quickly

Examples:

- Each group makes up a n -dimensional vector. Then each group decides whether the collection is linearly independent.

4. Board Work

- Helpful for solving multi-step problems, compare related problems

Examples:

- Several triple integrals but order of integration different. Does the order make a difference?
- Each group creates a 2×2 matrix, exchange, find eigenvalues

Cooperative Strategies for Day-to-day Usage contd.

5. Roundtable

- Each group gets only one worksheet. Each student adds the next line and passes it long. They may not comment on previous responses (unless this is a review of a topic) until the end. Encourages listening skills.

Examples:

- Write examples of function whose derivatives require chain rule. Specify f and g .

4. Minute Papers (use Poll-Everywhere to collect response)

- Obtain a quick snapshot of current beliefs and understandings.

Examples:

- List three ways Hyperbolic Geometry differs from Euclidean.

Cooperative Strategies for Day-to-day Usage contd.

7. Group Critique

- Necessary to explain constructive criticism. Post examples of helpful critiques.

Examples:

- Prove that every third Fibonacci number is even. Pass your paper to another group. Critique the proof you receive.

8. Triptych

- Specifically effective in studying a process and its reversal, e.g. differentiation.

9. Send a Problem

- Useful to review before an exam.

Cooperative Strategies for Day-to-day Usage contd.

10. Jigsaw

- Students initially in home groups. A multi-component problem is posed to class.
- Expert groups are formed, each contains exactly one student from each home group. Each expert group addresses one component of the problem.
- Students return to their home group. Each one teaches the other members about their portion.

Usage:

- Consider various cases in the proof of a theorem
- Study a topic outside the syllabus

Examples:

- After the class was taught Euler's formula, each group is assigned a regular polygon and asked to decide if there is a regular polyhedra.
- Each group is assigned two or more properties to verify to show that a collection of vectors forms a vector space.

Designing a Good Group Task

Designing a Good Group Problem

- Can be solved in more than one way
- Have multiple correct answers → e.g. give two 3d vectors that are skew
- Lead to follow-up questions (or more sophisticated ideas)
- Include a variety of operations that can be divided among the members
- With some not-immediately obvious feature → challenging, but comprehensible
- Is exciting and thought-provoking → avoid questions that students have seen before. However, still closely related to the lecture material
- Is Applicable → Most high school students have not seen the wide applicability of math to real-world phenomena. CL puts mathematics into perspective → provides challenges that directly apply the theory → motivates students to learn the theory → also helps them to develop the important skill of interpreting mathematical results.
- Is an Exploration or Review problem

A Group is Stuck, now what?

- **Listen to students. Ask them to walk you through their reasoning.**
 - Why did you start here?
 - How did you get that answer?
 - You made a choice here. Why did you make this choice? Why not that choice?
- **Ask questions to make sure all members have participated.**
 - Does everyone agree?
 - What thoughts does everyone have on that?
 - Does anyone want to share a different idea or approach?
- **Ask leading questions when a group is stuck.**
- **Refer students back to their resources.**
 - Did you have a chance to read/watch ____? Does anyone have notes on that?
 - The homework has a similar question to this. How did you answer that one?

They are no longer stuck, now what?

- **Visit another group and return later to assess the team's progress.**
- **Probe that students truly understand what they are doing by asking follow up questions.**
- **Ask students “why” about their process regardless if their answer is right or wrong!**

Designing Group Assessments

Design Assignments to promote CL

- CL is productive when GW is included directly in the assessment and grading process.
- Formative Assessment Options:
 - Informal observations, oral responses, ungraded homework
 - Use technology → Email, video blog, One-slide-posters
- Not all skills students learn can be assessed in a time-constrained setting, e.g. “speaking” or “evaluating” Math.
- Need to think of ways to structure class activities so that the motivation to collaborate is built directly into the task itself.
 - Design in a way so that if the work is divided, significant opportunities for learning are embedded in each component.

Some suggested group assessments and their challenges

| Element | Possible Benefits | Challenges |
|-------------------------|---|---|
| Journal | Individual reflection, metacognition | Time consuming to appraise, have to be done regularly |
| Participation grade | Motivate active involvement | Grade inflation |
| Group Task in Homework | Potential to cover complicated tasks | Tend to divide up the work, false sense of understanding |
| Group part in Quiz/Test | Provide group reward | Conflict over solution, does not measure individual achievement |
| Group lab/project | Motivation, accomplish more, learn from each other, | Freeloaders, logistics, multiple authors |

Sprinkle in some Metacognition

- Daily/Weekly Feedback
 - What main ideas, concepts, techniques have you learned? How have you contributed? How well do you feel you understand them? What are remaining issues that are unclear?
 - Grade on a -,0,+ scale.
- During Group Problem Solving
 - How did you/your team break down problems? Why does this answer make sense and how can you verify your work?
 - How does this relate to former problems/content/problem solving processes?
 - As you group worked, what roles did each person take (was there a leader? who did library/content research? Was someone primarily responsible for writing? What other roles?)
- After Team Problem Solving
 - Compare/contrast what you knew before, what tools/skills you applied
 - What is something you came in today not having understood, and what changed about your understanding? How?
- Group Project
 - Did you collaborate with other groups? Was there any benefit to this?
 - What did you contribute?
 - If you had 500 points to distribute, what would you suggest? Why?
 - Students reluctant to report negatively, but willing to reward a student with more points as an affirmation.

Thank you.

Sprinkle in some tasks that improve Metacognition

- A type of self-reflection
 - Think about how you think
 - Learn about how you learn
 - Develop cognitive self-awareness
 - Understand how to use different strategies in different situations
- Includes
 - Identify best strategies for learning
 - Monitoring plans throughout the process and revising them when they are not working
 - Choosing which learning strategy to employ and when
 - Self-evaluating a completed plan
- Goal
 - Transfer their learning to new scenarios
 - Actively tap into existing network of knowledge and connect the dots

References

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