Math 279: Putnam Seminar

Fall 2022 Syllabus

What is the Putnam Competition?

The William Lowell Putnam Competition is a challenging math competition administered to about 4000 undergraduates each year. The problems in the competition do not necessarily fall into typical categories covered by standard Math courses in college, and often challenge the students to think in innovative and creative ways.

What is Math 27901?

The Putnam Competition merely provides us an excuse to run a problem-guided tour of mathematics, while also developing core problem-solving skills that enhance the ability to learn and use higher mathematics. The questions which we will discuss in class are specially selected to spark discussions about famous mathematical results.

Unofficially, I like to think of this class as a *Problem-Solving Seminar*. Anything that qualifies as a *logic* or *math* puzzle, also qualifies as a *Problem* for this class. So, while we will use questions from previous Putnam exams as examples, we will also talk about *big picture* ideas like how to set the correct mindset for solving *hard* math problems in general. My aim is two-fold:

- To make you realize that 'solving a problem' is not the same as 'understanding the solution of a problem'. This, in particular, is important for both a student and a teacher, as you may often feel a disconnect between us regarding this very issue. After working out a solution (or if you know it beforehand), it may seem doable/easy in retrospect; but the hardest part often is how to start the process. In fact, for a problem that usually appears in a competition such as Putnam (or the Math Olympiad), it isn't often clear what, if any, theorem you learn during college is going to be needed. **So, our primary aim during the seminar would not be trying to prove/list a bunch of theorems; but rather to discuss some strategies which would help us decide what part of Mathematics to use to *start* solving a problem** (for example, 'trying the result in simpler cases' or 'drawing a picture' or 'shifting your perspective' etc.)
- To practice how to persevere with a difficult question, even when no goal might immediately be within your sights. This is the quintessential aspect of Mathematical scholarship. Whether you plan to go to grad school or industry after college, you will always face problems that are unknown to you. Your success will depend on whether, and how long you **stick with it**!

I will link two videos on two interesting problems from one of my favorite Math YouTube channels below, which give perfect examples of how to *start* thinking about a hard problem.

- The hardest problem on the hardest test
- The unexpectedly hard windmill question

You may want to watch these to get some motivation!

Course Credit, Time, and Location

The Putnam Seminar (Math 279) is a quarter-credit course graded S/NC. Your course grade will depend entirely on in-class attendance and participation, and not on your actual performance in Putnam. Note that **no prior math competition experience is required** (although familiarity with topics from Math 215 or Math 130 is useful), and beginners are welcome; you can also freely attend any of the sessions without having registered for the course!

We will meet once a week, Monday 3:00-3:50PM in Taylor 209.

Course components and Grading

There are no exams or projects in this course. There will be weekly reading exercises and optional practice problems as homework. You are allowed to freely collaborate with others and/or use the internet as a resource! Every week, I will ask you to write up a full formal solution for one or two problems we briefly discuss in class and submit it at the beginning of next class. **One firm expectation is that you will take the Putnam exam in December.** Notice that I only said 'take', there is no minimum passing grade that you must score.

Late Submission Policy

There is no penalty for late homework, except that all homework must be submitted by two weeks after its original due date, and no later than the final class day of the semester. However, students are strongly encouraged to submit homework on time, because this keeps the class in sync for healthy collaboration on homework, and the homework is very helpful for Putnam preparation.

Final grade

Your final grade will be calculated through the following formula:

Score = (1 for attendance + 1 for solution write-up + 1 for in-class participation) × (14 weekly sessions) + (9 point for attempting each half of the Putnam competition) × 2 halves

You will get an S if you score at least 45 (this can be negotiated based on exigent circumstances).

Textbook

There is no official textbook for this course. We will roughly follow the order of topics from the book **Putnam and Beyond**, by Razvan Gelca and Titu Andreescu. You are not required to buy the book; it is available for free through the College of Wooster library. I will also post a pdf version directly to the Moodle page.

The second resource is The Putnam Archive (kskedlaya.org), a website maintained by Prof. Kiran Kedlaya. This website lists all the Putnam problems and solutions from the last ~35 years!

Course Goals and Syllabus

By the end of the course, students should develop fundamental problem-solving skills, and become accustomed to concentrating on a problem for an extended period of time. Students should also be able to recognize when proofs are written with sufficient rigor and should gather a greater appreciation for mathematics as a broad field.

Over the weeks leading up to the Putnam exam, we plan to cover **a subset of** the following topics:

1. **New Problem-Solving Strategies**
    a. **Invariance principle** – problems where a proof depends on finding a mathematical quantity (e.g., an algebraic expression) or a property (e.g., monotonicity, concavity, parity etc.) that remains unchanged.
    b. **Coloring principle** – problems where you can find an invariant by coloring the plane!
    c. **Extremal principle** – problems (usually a proof by contradiction) that utilizes the smallest or largest number (or element of a set) that has certain properties.
    d. **Pigeon-Hole Principle** – A simple version can be stated as "if we put (n+1) pigeons into n holes, then at least 1 hole will have more than 1 pigeon".
2. **Polynomials** – Divisibility, Relation between roots and coefficients
3. **Number Theory** – Modular Arithmetic, CRT, Diophantine Equations
4. **Calculus** – Mean Value Theorem, Taylor Series
5. **Functional Equations** – Cauchy equations, the Triple iterate
6. **Inequalities** – AM-GM-HM, Cauchy-Schwarz, Jensen, Optimization – Lagrange multipliers
7. **Sequence** – Monotonicity, Convergence
8. **Recursion** – Solving linear homogeneous recurrence, proof by induction, generating functions
9. **Linear Algebra** – Matrix multiplication
10. **Combinatorics** – Permutation, combination, graph theory, probability
11. **Geometry** – Lines, planes, polygons, convexity, trigonometry