MATH 279: PUTNAM SEMINAR

FALL 2021 SYLLABUS

A. Course Credit

Math 27901 is a quarter-credit course. I will expect regular attendance from the students taking it for credit.

B. Course Time and Location

Monday 3:00-3:50PM in Taylor 209.

C. Course Goals

Officially the Math 27901 course is titled the Putnam Seminar. The Putnam is a challenging math competition administered to about 4000 undergraduates each year. The problems in the competition do not necessarily fall into typical categories covered by standard Math courses in college, and often challenge the students to think in innovative and creative ways. The course description for Math 279 in the college catalog can be paraphrased as "preparation for the William Lowell Putnam Mathematical Competition". However, I would like to clarify that this course is much more than that. Unofficially, I like to think of this class as a "Problem Solving Seminar". What kind of problems, you might ask? Anything that qualifies as a 'logic' or 'math' puzzle or question. So while we will use problems from previous Putnam exams as examples, we will also talk about how to set the correct mindset for solving 'hard' math problems in general. My aim is two-fold:

- To make you realize that 'solving a problem' is not the same as 'understanding the solution of a problem'. This in particular is important for both a student and a teacher, as you may often feel a disconnect between us regarding this very issue. After working out a solution (or if you know it beforehand), it may seem doable/easy in retrospect; but the hardest part often is how to start the solution. In fact, for a problem that usually appears in a competition such as Putnam (or the IMO), it isn't often clear what, if any, theorem you learn during college is going to be needed. *So our aim during our meetings wouldn't be trying to prove/list a bunch of theorems; but rather to discuss some strategies which would help us decide what part of Mathematics to use to "start" solving a problem* (for example, 'trying the result in simpler cases' or 'drawing a picture' or 'shifting your perspective' etc.)
- To practice how to persevere with a difficult question, even when no goal might immediately be within your sights. This is the quintessential aspect of Mathematical scholarship. Whether you plan to go to grad school or industry

after college, you will always face problems that are unknown to you. Your success will depend on whether, and how long you "stick with it"!

I will link two videos on two interesting problems from one of my favorite Math youtube channels below, which give perfect examples of how to "start" thinking about a hard problem.

- The hardest problem on the hardest test
- The unexpectedly hard windmill question

You may want to watch these to get some motivation!

D. Learning Objectives

Besides the Putnam Archive, we will be also using various problem-solving resources I have collected over the years, that will be linked through Moodle. You do not need to buy any book for this course, all you need is to have an inquisitive mindset. Here are some of the problem-solving strategies I plan to cover in the weeks leading up to Putnam. I have also included some very easy sample problems to give you an idea of what I am talking about.

- **The invariance principle** – problems where a proof depends on finding a mathematical quantity (e.g. an algebraic expression) or a property (e.g. monotonicity, concavity, parity etc.) that remains unchanged.
    - **Sample problem:** Start with an 8 × 8 chessboard with the usual coloring. In each move, you may repaint all squares of a 2 × 2 square (to the same color). The goal is to attain just one black square. Can you reach the goal?
- **The Extremal principle** – problems (usually a proof by contradiction) that utilizes the smallest or largest number (or element of a set) that has certain properties.
    - **Sample problem:** Is there a smallest positive real number?
- **Pigeon hole principle** – A simple version can be stated as "if we put (n+1) pigeons into n holes, then at least 1 hole will have more than 1 pigeon".
    - **Sample problem:** What is the minimum number of students you need to ensure that at least three of them have birthdays on the same day of the week?
- **Popular Sequences and series** – We are not proving convergence/divergence here. We will instead look at some famous sequences such as the Fibonacci sequence, Chebyshev polynomials, etc., and learn how to utilize Mathematical Induction.
- **Inequalities and Optimization** – Inequalities show up in all fields of Math, both pure and applied, whenever we try to optimize a quantity. We will talk about AM-GM-HM, Cauchy-Schwartz, inequalities utilizing convex functions, etc.

Students who have taken multivariable will also be able to recognize optimizations using the Lagrange multiplier.

- **Modular arithmetic and Solving Diophantine equations** – Existence problems that ask you to find some integers a,b,c (or prove that they can't be found) satisfying a given equation. This often involves looking at the remainder of the terms when divided by some other number!
    - **Sample problem:** Find all pairs of prime numbers p and q such that p^2 = 2 q^2 + 1.
- **Combinatorics leading to questions involving probability** – This will involve some easy counting techniques.

E. Course components

There are no exams or projects in this course. There might be occasional short reading exercises or puzzles to think about at home. **The only expectation is that you will take the Putnam exam in December.** Notice that I only said 'take', there is no 'minimum' passing grade that you must score. Your course grade (S/NC) will depend entirely on in-class attendance and participation.